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Estimating the Probability of Crashing for a Terrain-Following Missile

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The probability of crashing (P_c) is an important measure by which a mission planner assesses, in advance, the chances of success of a terrain-following missile. The altitude clearance command is programmed accordingly. Monte Carlo methods are unreliable for estimating small values of P_c over a typical mission length, and require extensive simulation to improve this reliability. An analytic method for estimating P_c is described in this paper. This uses an analytical expression for mean frequency of crashing, in conjunction with the statistics of missile altitude error and error rate obtained from a simulated terrain-following flight over a stretch of terrain much shorter than that needed for Monte Carlo runs. The method is applicable to both Gaussian and non-Gaussian error statistics, and it provides data on small P_c values for typical mission lengths. This information would be much more expensive to obtain by Monte Carlo techniques.

Nomenclature

A	= event of a positive crossing of level h_0, event of a crash
a	= parameter of first-order autoregressive process
$E(t),e(t)$	= missile altitude error random process, sample function
$E'(t),e'(t)$	= missile altitude error rate random process, sample function
$\langle \lvert E'(t)\rvert \rangle$	= ensemble mean of absolute value of error rate
$f_{EE'}(e,e')$	= joint probability density function of altitude error and error rate
$f_E,f_{E'},f_{CL}$	= probability density function of altitude error, error rate, clearance altitude
$\langle \lvert H'_{CL}(t)\rvert \rangle, \overline{\lvert h'_{CL}\rvert}$	= ensemble mean, sample mean of absolute value of clearance rate
$H_{CL}(t),H_{CL'}(t), h_{CL}(t),h_{CL'}(t)$	= missile clearance altitude, altitude rate random process, sample function
$h(t)$	= missile inertial altitude
$H_T(t),h_T(t)$	= terrain random process, sample function
h_0	= clearance command to altitude control system
l_1,l_2	= confidence limits for μ_{CL}
l_3,l_4	= confidence limits for σ_{CL}
l_5,l_6	= confidence limits for $\sigma_{CL'}$
L_c	= length of independent interval
N	= number of clearance, clearance rate samples
n,n_{mc}	= number of independent samples in flight time T, T_{mc}
n_c	= number of crashes
$P_c,\hat{P}_c$	= true probability of crashing, estimate of P_c
R^2,r^2	= sample variance estimator, estimate of clearance rate
S^2,s^2	= sample variance estimator, estimate of clearance
T,T_{mc}	= total flight time or range, total length of Monte Carlo flights (in seconds or nautical miles)
x_i,y_i	= samples of clearance, clearance rate
V	= missile speed
V_i	= input white noise to autoregressive process
$\bar{X},\bar{x}$	= sample mean estimator, estimate of clearance
$\bar{Y},\bar{y}$	= sample mean estimator, estimate of clearance rate
Z_i	= autoregressive process
$1-\alpha$	= level of confidence in estimates
$\beta,\gamma,\epsilon,\delta$	= parameters in Johnson curve fit
$\lambda,\hat{\lambda}$	= mean frequency of crashing (per second or per nautical mile), estimate of same
$\mu_e,\mu_{e'},\mu_{CL}$	= true mean of altitude error, error rate, clearance
$\sigma_e,\sigma_{e'},\sigma_{CL},\sigma_{CL'}$	= true standard deviation of altitude error, error rate, clearance, clearance rate
$\chi_{1-\alpha/2,2n_c}$	= $(1-\alpha/2)$ point for chi-square variable with $2n_c$ degrees of freedom
$\nu=\lambda T_{mc}$	= Poisson parameter

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Introduction

A LOW-FLYING missile in a terrain-following mode typically uses a radar altimeter to sense its clearance altitude above the ground, since a forward-looking radar would increase its chances of being detected. One of the main tasks confronting the mission planner is to set, in advance, the clearance command inputs to the missile altitude control system. These settings will depend on the nature of the terrain being overflown; they should be as low as possible to avoid detection, and yet not so low that the missile crashes. The task is complicated by the fact that, due to lateral navigational errors, the exact terrain to be overflown is not known in advance, even if relatively error-free maps of the mission route were available.

An important criterion, which enables the planner to assess the missile's projected performance and to program reasonable clearance commands, is P_c, the probability of crashing (or "clobber") over a certain range or flight time. Many terrain-following studies rely on Monte Carlo simulation to estimate P_c over various terrain types. In Ref. 1, it is shown that these methods are unreliable for estimating small P_c values over a reasonable length of mission.

In contrast to Monte Carlo methods, which count the numbers of observed crashes, an analytic method for deriving P_c was developed in Ref. 2. For brevity, in this paper the technique is called the "analytic method," although strictly speaking, it is semianalytic in that it uses an analytical expression for the mean frequency of crashing, λ, in conjunction with the statistics of missile altitude error, and error rate obtained from a simulated terrain-following flight over a stretch of terrain much shorter than that needed for Monte Carlo runs. An expression for P_c is given in Ref. 2 for the special case where the underlying altitude error process is Gaussian. In many cases in practice, however, due to unsymmetrical climb and dive limits, and other nonlinearities in the altitude control system, the clearance error distribution is skewed so that the Gaussian assumption is no longer valid.

In this paper, the method of Ref. 2 is extended to the situation of non-Gaussian error statistics, and analytical expressions for P_c are given for various assumptions on the relationship between the error and error rate processes. The use of these expressions to estimate P_c for both Gaussian and non-Gaussian statistics is indicated. Confidence limits for the true P_c, based on the estimate, are derived for the Gaussian case. Confidence limits are also obtained for the Monte Carlo estimate of P_c based on a Poisson distribution of crash events, to complement the confidence limits for Monte Carlo estimates of P_c based on independent trials.[1]

Finally, P_c estimates obtained from Monte Carlo simulation, and by the analytic method of this paper, are compared. Results indicate reasonable agreement between the two methods at clearance commands, which give large values of P_c. However, while the analytic method gives estimates of small P_c values which vary smoothly with increasing clearance command, the Monte Carlo method provides little information in this situation. Furthermore these results are provided from much shorter simulation lengths than required for the Monte Carlo approach, resulting in considerable savings in computer time.

Analytical Expressions for Mean Frequency of Crashing

The missile, flying at constant speed, is assumed to be controlled to a certain clearance altitude h_0 as indicated in Fig. 1. Because of navigational errors, it is not known precisely in advance which particular stretch of terrain will be overflown. Hence, the terrain input $h_T(t)$ may be considered as a realization of a random process $H_T(t)$, and the resulting altitude error $E(t)$ is also a random process with realization $e(t)$ as shown in Fig. 2a.

The missile hits the ground when $e(t)$ crosses the h_0 line with $e'(t)>0$. Each such crossing is considered to be an event A as depicted in Fig. 2b. If the mean frequency λ of such events is known, then, assuming a Poisson distribution of event points,[2] the probability of crashing or "clobber" may be computed as

$$P_c = 1 - \exp(-\lambda T) \tag{1}$$

where T is the flight time or range.

Under the assumption that $E(t)$ is stationary, classical "zero crossing" theory[3,4] gives the mean frequency as

$$\lambda(h_0) = \int_0^\infty e' f_{EE'}(h_0, e')\,\mathrm{d}e' \tag{2}$$

This is the most general expression for λ under the stationarity assumption. It can be simplified if certain other assumptions are valid. First, it may be noted that a stationary random process, and its derivative, are uncorrelated at the *same* time t.[4] This does not necessarily imply independence of $E(t)$, unless they are Gaussian. Suppose, however, that $E(t)$ and $E'(t)$ are independent, with arbitrary distributions $f_E(e)$ and $f_{E'}(e')$. Then

$$f_{EE'}(e,e') = f_E(e) f_{E'}(e') \tag{3}$$

and Eq. (2) becomes

$$\lambda(h_0) = f_E(h_0)\int_0^\infty e' f_{E'}(e')\,\mathrm{d}e' \tag{4}$$

For any stationary differentiable process, the mean of the derivative process is zero, i.e., $\mu_{e'}=0$. If $f_{E'}(e')$ is symmetrical with respect to its mean, Eq. (4) becomes

$$\lambda(h_0) = \tfrac{1}{2} f_E(h_0)\langle |E'(t)|\rangle \tag{5}$$

Finally, if both $E(t)$ and $E'(t)$ are Gaussian,

$$\lambda(h_0) = \frac{1}{2\pi}\frac{\sigma_{e'}}{\sigma_e}\exp\left[-\frac{(h_0-\mu_e)^2}{2\sigma_e^2}\right] \tag{6}$$

This reduces to the expression for λ given in Eq. (8) of Ref. 2 if $\mu_e = 0$.

Since, from Fig. 1b,

$$E(t) = h_0 - H_{CL}(t) \tag{7}$$

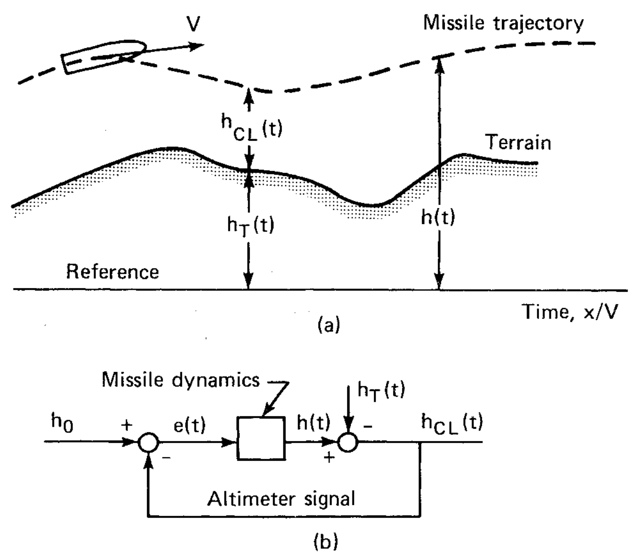

Fig. 1 Terrain following.

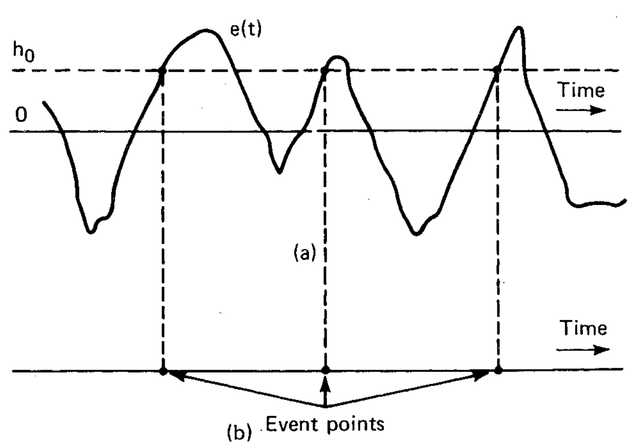

Fig. 2 Crossing of arbitrary level by altitude error signal.

it follows that Eq. (5) may be written in terms of the distribution for clearance altitude as

$$\lambda(h_0) = \tfrac{1}{2} f_{CL}(0) \langle |H'_{CL}(t)| \rangle \tag{8}$$

while Eq. (6) becomes

$$\lambda(h_0) = \frac{1}{2\pi}\frac{\sigma_{CL'}}{\sigma_{CL}}\exp\left[-\frac{\mu_{CL}^2}{2\sigma_{CL}^2}\right] \tag{9}$$

The appropriate one of the above expressions for λ can be inserted in Eq. (1) to obtain P_c. Of course, in general, the exact probability distributions required will not be known. An estimate of λ can be obtained from the statistics resulting from simulation of a terrain-following flight over a single terrain strip, as described in the following section. Which of the expressions for λ is appropriate, can be determined from inspection of the histograms for error and error rate (or clearance and clearance rate).

Estimating P_c from Curtailed Clearance Statistics

As noted above, Eq. (2) is the most general expression for λ, assuming stationarity. If nothing further is known about the joint probability density of $E(t)$ and $E'(t)$ then, in principle, an estimate of λ may be obtained as follows:

1) Simulate a missile flight over a suitable terrain sample with constant clearance command h_0.
2) From the simulation, measure $e(t)$ and $e'(t)$ (or $h_{CL}(t)$ and $h'_{CL}(t)$).
3) Estimate the lower order joint central moments (say, up to the fourth moment) of $E(t)$ and $E'(t)$ based on the observed samples.
4) Fit a joint density function to the data using the moment estimates as constraints.
5) Insert this fitted expression with $e = h_0$ for $f_{EE'}(h_0, e')$ in Eq. (2).
6) Numerically integrate the resulting expression to obtain an estimate of $\lambda(h_0)$.

Note that a curve fit is necessary, rather than directly integrating the joint histogram for $e(t)$ and $e'(t)$, since there will be no points with $e = h_0$ in the histogram unless the missile hits the ground.

Although the above procedure is feasible in principle, it presents a formidable task in practice because of the difficulty of fitting a joint density function to the estimated moments. Fortunately, however, tests for independence[5] carried out on simultaneous samples of e and e' taken from the simulation indicated that, in general, $E(t)$ and $E'(t)$ may be considered to be independent at the same time t. Furthermore, it was noted that, in many cases, the error or clearance histogram was skewed considerably (see Fig. 3) due to the unsymmetrical nonlinearities in the altitude control loop, whereas the error rate was symmetrical with approximately zero mean. In such cases, it is appropriate to use expressions in Eq. (5) or Eq. (8) for λ.

An estimate of the means of the absolute value of clearance rate required in Eq. (8) may be obtained from the clearance rate samples as

$$\overline{|h'_{CL}|} = \frac{1}{N}\sum_{i=1}^{N} |y_i| \tag{10}$$

Estimating $f_{CL}(0)$ from the clearance samples requires that the latter be fitted with a probability density function. N.L. Johnson[6] outlined a method for fitting a probability density function to experimental data. The method uses the first four moments of the data as constraints. Three families of distributions are available for fitting, namely, unbounded, bounded, and log-normal. The data is classified accordingly.

The method was coded for use in A Programming Language (APL) by J.N. Bramhall.[7] The clearance data from each simulated flight were stored on disk for use with this program. The latter computed the parameters of the distribution fit and the value of the function at zero, which is required for the λ estimate. A typical example of a fitted Johnsonian function is shown overlaid on the clearance histogram in Fig. 3. Note that the histogram itself is not fitted; its shape is not unique since it depends on the bin size. Instead, as mentioned above, a fit is made to the first four moments of the data.

Using the values of $f_{CL}(0)$ so obtained, the estimate of λ is computed according to Eq. (8). The probability of crashing estimate for any desirable time of flight or distance can be computed from Eq. (1); this P_c value is, of course, for the h_0 value selected for the simulated flight.

It should be noted that the actual choice of h_0 for the simulated run has little effect on the clearance statistics upon which the probability of crashing is based. For example, if one run is made at $h_0 = h_1$, and a second run over the same strip at $h_0 = h_2$, the mean clearance will change by an amount $(h_2 - h_1)$, but the standard deviation of clearance σ_{CL}, and the mean of the absolute value of clearance rate $\langle |h'_{CL}(t)| \rangle$, will be relatively unchanged, except for some minor effect due to change in air density if the difference between h_1 and h_2 is large. The histogram of clearance values will have the same shape, but will be translated by the change in mean value. This relative invariance in clearance statistics for different h_0, over a given terrain strip, permits the required clearance command to be determined for a desired P_c, as will be seen below.

Suppose that it is desired to find the clearance command required to give a specified P_c over a specified range T of terrain similar to that overflown in the simulation. Combining Eqs. (1) and (8), substituting $\overline{|h'_{CL}|}$ gives

$$f_{CL}(0) = -2\ln(1-P_c)\,(T\overline{|h'_{CL}|}) \tag{11}$$

The terms on the right side of this equation are known. The value of $f_{CL}(0)$ so obtained will not agree with that obtained from the data-fitting procedure since, in general, the simulated flight was not made at the proper h_0. It is therefore necessary to raise or lower h_0 accordingly. The amount by which h_0 must be changed is determined as follows:

Suppose that an unbounded distribution is fitted to the test data. Then, from Ref. 7

$$f_{CL}(x) = \frac{\delta}{\sqrt{2\pi}}\exp\frac{(-\tfrac{1}{2}\{\gamma+\delta\ln[B+(B^2+1)^{1/2}]\}^2)}{\beta(B^2+1)^{1/2}} \tag{12}$$

where $B = (x-\epsilon)/\beta$. The values of the four parameters β, γ, δ, and ϵ are given by the data-fit program. For $x = 0$, Eq. (12) gives the $f_{CL}(0)$ value corresponding to the flight h_0. The

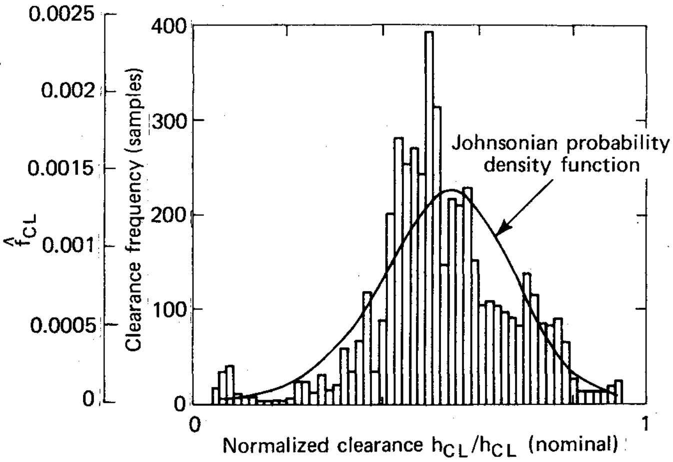

Fig. 3 Johnsonian probability density function overlaid on clearance histogram.

equation was programmed on a hand calculator. With the given parametric values, x was varied to obtain a value of $f_{CL}(x)$ equal to that required by Eq. (11). About five or six iterations were usually required to find the correct value of x. The latter is the amount by which the command altitude must be changed to obtain the specified P_c value. The basis for the foregoing procedure is the above mentioned relative invariance of clearance statistics as h_0 is varied.

The problem of estimating P_c is simplified considerably if the error and error-rate statistics are approximately Gaussian. In that case, Eq. (6) or (9) may be used with, for example, sample estimates of σ_{CL}, $\sigma_{CL'}$, and μ_{CL} replacing those parameters in Eq. (9).

Confidence Limits for P_c Estimates Using Analytic Method

Consider first, the case where the statistics may be assumed to be approximately Gaussian. Then, an estimate of λ based on Eq. (9) is

$$\hat{\lambda}(h_0) = \frac{1}{2\pi}\frac{r}{s}\exp\left(-\frac{\bar{x}^2}{2s^2}\right) \tag{13}$$

where

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^2 \tag{14}$$

$$r^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{y})^2 \tag{15}$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N}x_i \qquad \bar{y} = \frac{1}{N}\sum_{i=1}^{N}y_i \tag{16}$$

and x_i, y_i, $i=1, N$ are samples of clearance and clearance rate respectively, measured from a simulated terrain-following run over a single terrain strip. If the samples are taken far enough apart to be independent, then the estimators S^2 and R^2 corresponding to Eqs. (14) and (15) are chi-square distributed with $N-1$ degrees-of-freedom, while random variables $\bar{X}$ and $\bar{Y}$ have a student-t distribution with $N-1$ degrees-of-freedom. Both the chi-square and the student distributions approach the Gaussian for $N>100$, which generally will be true in practice. Using the sampling distributions, $(1-\alpha)$ confidence limits can be determined about $\bar{x}$, s, and r for the true parameters μ_{CL}, σ_{CL}, and σ'_{CL}. The corresponding simultaneous confidence interval[8] is $(1-3\alpha)$, e.g., if the individual confidence intervals are each 95% for the three parameters, the simultaneous confidence interval is 85%. Thus, if l_1 and l_2 are the $(1-\alpha)$ confidence limits for μ_{CL}, l_3 and l_4 are the limits for σ_{CL} and l_5 and l_6 are the limits for σ'_{CL} then the $(1-3\alpha)$ confidence limits for λ are

$$\left[\frac{1}{2\pi}\frac{l_5}{l_3}\exp\left(-\frac{l_2^2}{2l_3^2}\right) \quad \frac{1}{2\pi}\frac{l_6}{l_4}\exp\left(-\frac{l_1^2}{2l_4^2}\right)\right] \tag{17}$$

These are obtained from Eq. (13) by inserting the appropriate limits of the estimates. Figure 4 illustrates how these limits, extrapolated to P_c via Eq. (1), vary with N for a case where the estimates $\bar{x}$, s, and r (taken from terrain following simulation) and the flight range T were selected to give a P_c estimate of 3%. It is evident from the figure that about 1000 independent samples are needed in this case before the true value of P_c is close to the estimate with the specified degree of confidence.

To obtain confidence intervals for an estimate of λ and P_c based on Eq. (8) is a difficult problem. The Johnson curve-fit routine mentioned above uses estimates of the first four moments to obtain estimates of skewness and kurtosis for the clearance distribution. Charlier[9] computed the variances of the skewness, kurtosis, and higher order measure estimators under the hypothesis that the distribution is approximately Gaussian. They are at most of the order $1/N$. Since Eq. (8) is used because of the pronounced non-Gaussian nature of the statistics, Charlier's results would have limited validity in this situation. What are required are sampling distributions for the skewness and kurtosis estimators for non-Gaussian statistics. This is a subject for further research.

Monte Carlo Estimate of P_c

The problem of estimating P_c from Monte Carlo simulation is treated in Ref. 1. Some additional results are presented here. Briefly, this method for P_c estimation involves simulation of a terrain-following flight at a fixed clearance command h_0 over a long stretch of terrain T_{mc} and counting the number of crashes n_c that occur. Then an estimate of λ is given by

$$\hat{\lambda}(h_0) = n_c(h_0)/T_{mc} \tag{18}$$

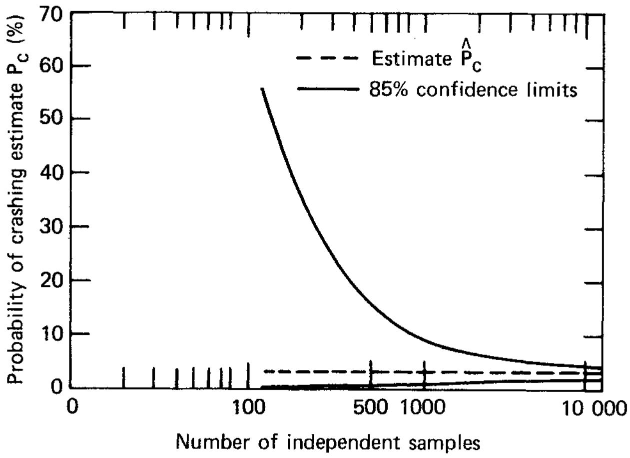

Fig. 4 85% confidence limits for P_c based on Gaussian statistics using analytic method.

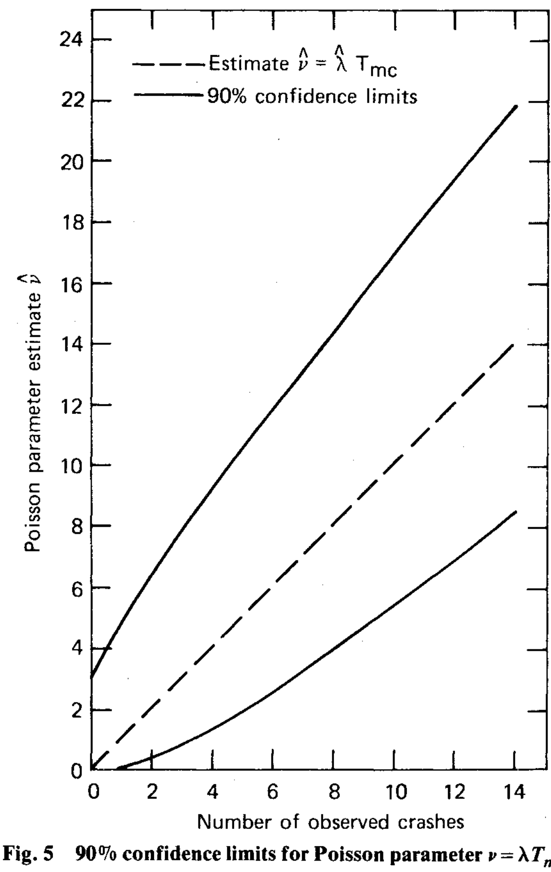

Fig. 5 90% confidence limits for Poisson parameter $\nu = \lambda T_{mc}$.

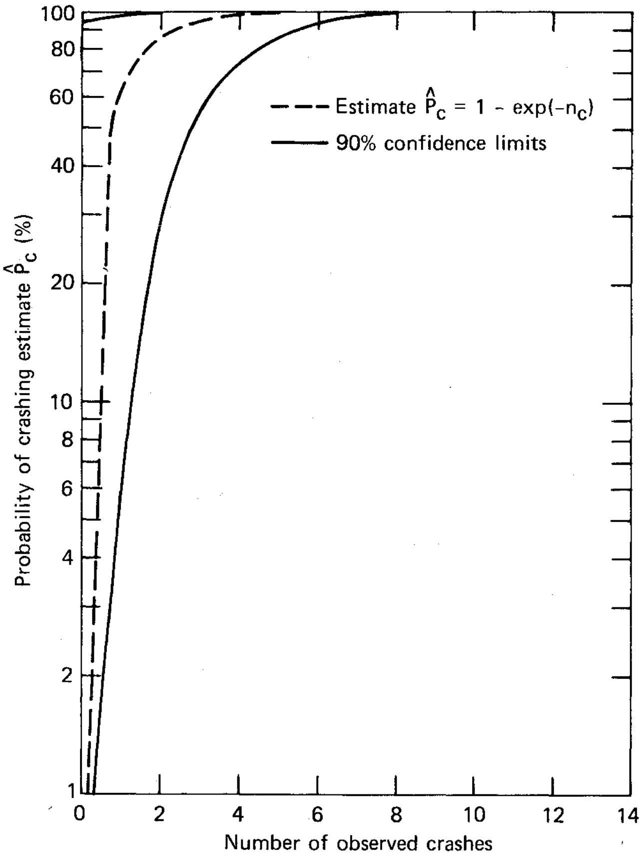

Fig. 6 90% confidence limits for P_c over a Monte Carlo length $T = T_{mc}$.

with probability of crashing estimate for an arbitrary flight time or range T as

$$\hat{P}_c(h_0,T) = 1 - \exp[-\hat{\lambda}(h_0)T] \tag{19}$$

Confidence limits for P_c based on Monte Carlo estimates were obtained in Ref. 1. These required the assumption that T_{mc} could be divided up into a number n_{mc} of independent intervals or Bernoulli trials. Then, an estimate of P_c over n such intervals is

$$\hat{P}_c(h_0,n) = 1 - (1 - n_c(h_0)/n_{mc})^n \tag{20}$$

While, in principle, $\hat{P}_c$ given by Eq. (20) approaches that given by Eq. (19) (with $T = T_{mc}$ and $n = n_{mc}$) as $n_{mc} \to \infty$ and the interval length $L_c \to 0$, T_{mc} remaining fixed, in practice it is not reasonable to let $L_c \to 0$, since this is on the order of the correlation length of the missile altitude error signal. It is to be expected then, that confidence limits for P_c based on an experiment that counts the number of independent intervals as implied by Eq. (20), will differ somewhat from limits based on a more direct estimate of λ as implied by Eq. (19). The latter limits may be obtained by noting that the $(1-\alpha)$ confidence interval for the Poisson parameter $\nu(=\lambda T_{mc})$ is[10]

$$[\tfrac{1}{2}\chi^2_{1-\alpha/2,2n_c},\ \tfrac{1}{2}\chi^2_{\alpha/2,2(n_c+1)}] \tag{21}$$

These limits are plotted for $\alpha = 0.1$ in Fig. 5 for up to 14 observed crashes. For convenience of display, smooth continuous curves are shown; in fact, because the number of crashes must be integer, stepwise-changing curves would be more correct. The corresponding 90% confidence limits for P_c are shown in Fig. 6 for a flight of length $T = T_{mc}$. This confirms the conclusion of Ref. 1; Monte Carlo methods are unreliable for estimating small values of P_c over a range corresponding to a reasonable length of mission. Monte Carlo simulation would be necessary over a range many times that of the mission.[1]

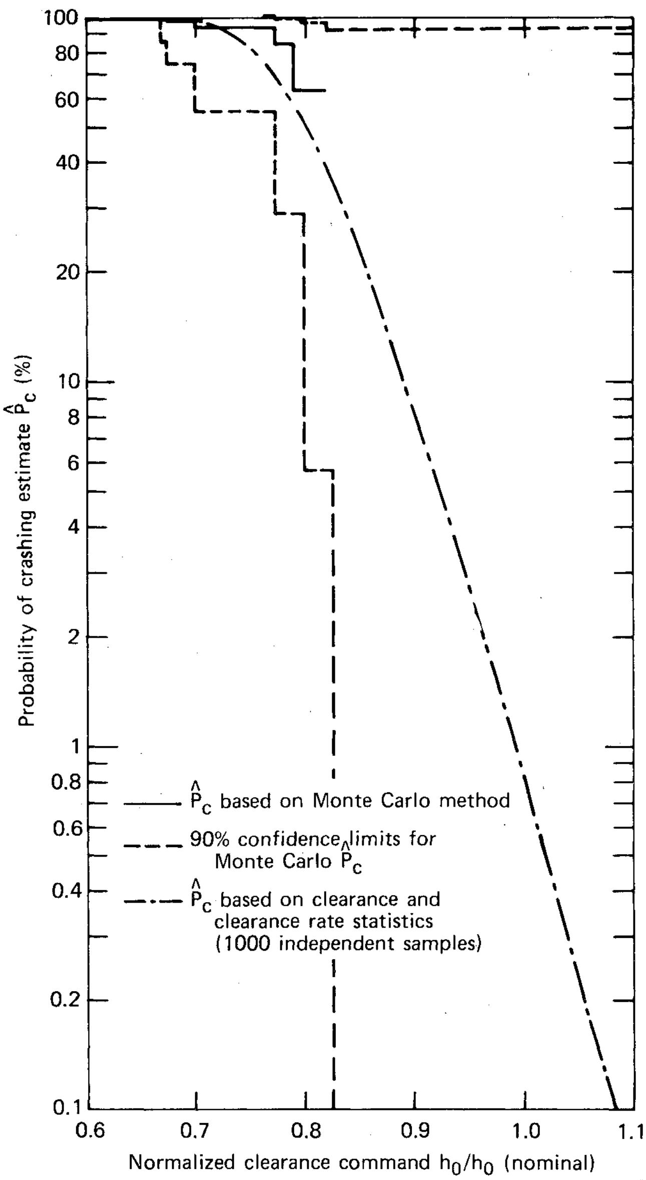

Fig. 7 Comparison of P_c estimates vs clearance command, over a Monte Carlo length $T = T_{mc}$.

Experimental Results

To compare the Monte Carlo and analytic methods for estimating P_c, a three degree-of-freedom digital simulation was made of a missile in terrain-following mode. To provide relatively homogeneous terrain data for the comparison, the terrain input was generated synthetically using a first-order autoregressive (AR) or Markov process[11]:

$$Z_i = aZ_{i-1} + V_i \tag{22}$$

where Z_i is the terrain random process in discrete time, and V_i is discrete white noise. The value of the parameter a and the variance of V_i were selected so that the resulting AR process would approximate actual terrain data in its correlation and frequency characteristics. The problem of terrain classification is treated in Ref. 12. A constant value for clearance command h_0 was selected, and the missile flown over a length of terrain T_{mc}, which was chosen to be on the order of a typical mission length. The number of crashes n_c in T_{mc} for the selected h_0 value was noted. The simulation also contained logic to count the number of crashes which would

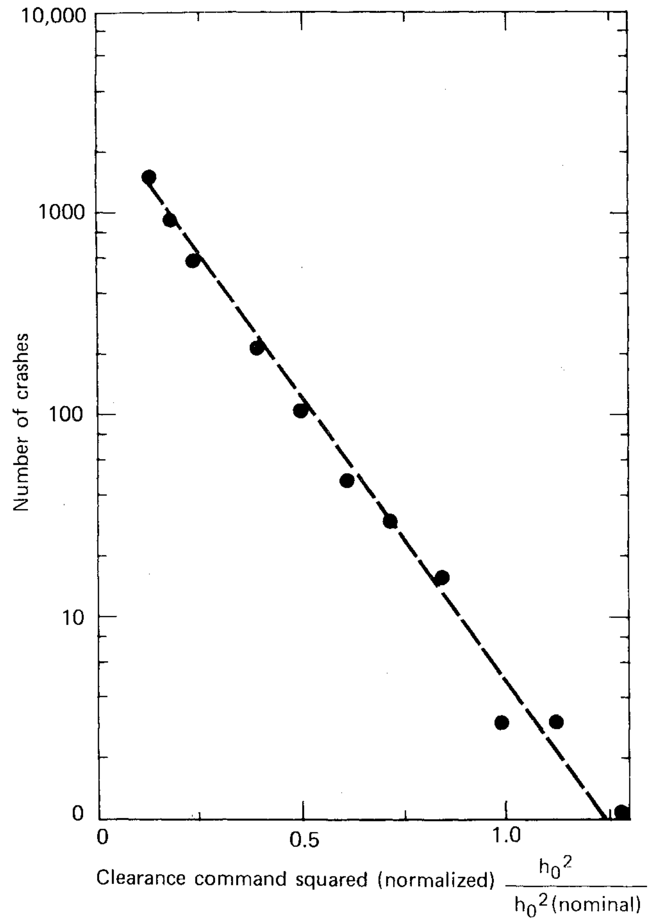

Fig. 8 Number of crashes vs clearance command squared from Monte Carlo simulation.

have occurred if h_0 were raised or lowered by specified increments. The P_c estimate resulting from this Monte Carlo approach was then computed by combining Eqs. (18) and (19) with $T=T_{mc}$, obtaining

$$\hat{P}_c = 1 - \exp(-n_c) \tag{23}$$

which has been plotted in Fig. 6. The estimate of P_c so obtained jumps from zero for no crashes counted to 63.2% for one crash, 86.5% for two crashes, etc. If one crash occurs for a certain h_0 then, raising h_0 by a few feet, say, might result in no crashes, causing the large change noted in $\hat{P}_c$. It is clear from Fig. 6, however, that little information is conveyed from the zero crash case; with 90% confidence, the true P_c can lie anywhere between zero and 95%!

In a typical experiment, the clearance and clearance rate statistics from 1000 independent samples (about 20% of T_{mc}) were tallied to give an estimate of P_c according to Eq. (13) extrapolated to T_{mc} via Eq. (1). This estimate, and that from the Monte Carlo method, are plotted vs normalized clearance command settings on Fig. 7. The confidence limits for the Monte Carlo estimate are also shown. It is evident that there is reasonable agreement between the two methods up to the clearance altitude at which no crashes occur; thereafter, little information is given by the Monte Carlo method, whereas the analytic method indicates a P_c estimate that decreases rapidly with increasing h_0. It also shows the clearance command setting for low value of $\hat{P}_c$ (say, 1 to 5%) for a mission of length T_{mc}. To get similar information from the Monte Carlo method would require that T_{mc} be increased to about 20 to 100 times the mission length (see Ref. 1), resulting in greatly increased computing costs, especially if a reasonably complete missile/autopilot is simulated.

Results similar to those shown in Fig. 7 were obtained when the missile climb-and-dive limits were made unequal, resulting in non-Gaussian clearance statistics comparable to Fig. 3 and necessitating an estimate of P_c based on Eq. (8).

A plot of n_c vs h_0^2 as shown in Fig. 8, indicates a relationship that is approximately exponential. Since $n_c = \hat{\lambda} T_{mc}$ and T_{mc} is fixed, this implies that $\hat{\lambda}$ varies exponentially with h_0^2 as predicted by Eq. (6).

Conclusions

1) The analytical technique presented here offers a practical way to estimate the probability of crashing, P_c, with much less simulation than that required for a Monte Carlo estimate.

2) Where results for the two approaches overlap, the Monte Carlo method tends to validate the analytic method.

3) Confidence intervals for P_c are readily obtainable in the case of the estimate provided by the analytic method with Gaussian statistics, and also for the Monte Carlo estimate. However, determination of the confidence interval for P_c based on non-Gaussian statistics requires further research.

4) Although the method described was developed specifically for the case of a terrain-following missile with downward looking radar, it seems applicable also to the similar problem of estimating P_c for a terrain-following missile or aircraft with forward-looking radar.

Acknowledgments

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